

Influence of long range Coulomb interaction on the electronic Mach-Zehnder interferometer of quantum Hall edge states

Hyun C. Lee

Department of Physics and Basic Science Research Institute, Sogang University, Seoul, Korea 121-742

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The influences of long range Coulomb interaction on Mach-Zehnder interferometer constructed on quantum Hall edge states is studied employing the bosonization method. The interaction of interchannel zero modes is shown to give rise to a characteristic energy scale which is of the order of the period of experimentally observed lobe pattern of visibility. The nonmonotonic behavior of visibility as found by Chalker *et al.* is understood analytically using the asymptotic analysis.

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I. INTRODUCTION

An electronic Mach-Zehnder interferometer (MZI) is a type of matter wave interferometer that has been realized with (integer) quantum Hall (QH) edge systems.¹⁻³ Among many interesting phenomena observed in electronic MZI, we focus on the puzzling lobe pattern of the interference visibility of differential conductance $\sigma(V)$ (V is a bias voltage).^{2,4}

This lobe pattern is hard to understand in the framework of noninteracting electrons so it is generally believed to be due to the many-body interaction. A few theoretical proposals have been made to explain this lobe pattern: the introduction of additional edge modes,^{5,6} the decoherence and finite temperature effect,⁷ and the shot-noise effects.⁸ The visibility of MZI of fractional QH edges which does not include the long range Coulomb interaction (LRCI) has been also studied recently.⁹

In this paper, we generalize the approach of Ref. 7 in two ways: (1) LRCI of the zero modes (see below) between two channels which comprise MZI is included and (2) both the integer QH states and the Laughlin fractional QH states at filling fraction $\nu=1/(2n+1)$, $n=1,2,\dots$ are considered. In the fractional case, we have to take both the fractional quasiparticle and the electron tunneling into account at point contacts.

A possible relevance of the interchannel LRCI may be argued as follows. The length scale of MZI in Ref. 2 is about $R \sim 5 \mu\text{m}$. Taking the typical dielectric constant of QH devices to be $\epsilon \sim 10$, the associated interchannel Coulomb energy scale E_c is $e^2/R\epsilon \sim 5 \times 10^{-2} \mu\text{eV}$, which is of the same order of magnitude as the observed period of the lobe pattern of visibility (see Figs. 3 and 4 in Ref. 2). This energy scale is well expected to depend on the geometry of MZI as well as the applied magnetic field. The investigation of this interchannel interaction of zero modes (defined below) requires a very careful treatment of the so-called Klein factors of bosonization formula, which constitutes the most significant part of this paper.

The results of Ref. 7, in particular, the nonmonotonic behavior of visibility as a function of bias voltage (Fig. 6 of Ref. 7), depend crucially on the asymptotic behavior of the one-electron Green's function in the presence of many-body interactions. We analyze the asymptotic behavior of the Green's function employing the method of asymptotic

analysis,¹⁰ thus providing more analytic understanding of the results. It turns out that the momentum dependence of the interaction matrix element plays an important role as we will discuss. The main results of this paper are (1) the exact time-dependent Klein factor which gives rise to the interchannel Coulomb energy scale E_c [Eq. (16)], (2) the tunneling current which incorporates the energy scale E_c [Eqs. (23) and (24)], and (3) the analytic form of the visibility (35).

II. MODEL OF MZI OF QH EDGES

The Hamiltonian which applies to both the integer and fractional Laughlin QH edge state and acts within each channel is^{11,12}

$$\hat{H}_{\text{intra}} = \sum_{i=1,2} \frac{\pi v_i}{\nu} \int dx [\rho_i(x)]^2 + \frac{1}{2} \int dx dy V(x-y) \sum_{i=1,2} \rho_i(x) \rho_i(y), \quad (1)$$

where $i=1,2$ is the channel index of MZI (see Fig. 1 of Ref. 7 for a schematic view of MZI). $\rho_i(x) = \frac{\hat{N}_i}{L_s} + \frac{1}{2\pi} \partial_x \phi_i$ is the density operator of edge i . \hat{N}_i is the number operator of edge i whose momentum is zero, hence it is often referred to as a zero-mode operator in the context of bosonization.¹³ ν is the filling fraction of QH system and L_s is the system size. ϕ_i is the boson operator which describes the collective harmonic modes with nonzero momentum. v_i is the velocity of collective modes of channel i . Two channels will be assumed to be identical so that $v_1 = v_2 \equiv v_0$. $V(x) = \frac{e^2}{\epsilon\sqrt{x^2+a^2}}$ is the LRCI acting within each channel. a is a short-distance cutoff. The fundamental commutation relation of density operators is¹¹ $[\rho_i(x), \rho_j(y)] = i\hbar \delta_{ij} \frac{\nu}{2\pi} \partial_x \delta(x-y)$.

There are two interactions which couple two channels $i=1,2$. One is the tunneling interaction at two quantum point contacts (a and b) which play the role of beam splitters of an optical interferometer,

$$\hat{H}_t = t_a \Psi_1^\dagger(0) \Psi_2(0) + t_b \Psi_1^\dagger(x=l_1) \Psi_2(x=l_2) + \text{H.c.}, \quad (2)$$

where the operator $\Psi_i(x)$ can be either electron or quasiparticle operator depending on the character of the point con-

tacts. The bosonized expression of these operators is given in Eqs. (7) and (11). The tunneling amplitudes depend on the enclosed flux Φ via $t_{ab}^* = |t_{ab}^*| e^{i\Phi/\Phi_0}$, where Φ_0 is the flux quantum.⁷

The other interaction which can couple two channels is the LRCI between two channels. For simplicity, we consider the interaction between zero modes only. This is because the interchannel LRCI becomes singular logarithmically in the zero-momentum limit (the singularity is cut by the finite system size). The Hamiltonian for the interchannel LRCI is taken to be

$$\hat{H}_C = E_C \hat{N}_1 \hat{N}_2. \quad (3)$$

Extracting the zero-mode parts from Eqs. (1) and (3), we can define a Hamiltonian for zero modes only,

$$\hat{H}_{\text{zero}} = \frac{\pi\nu_0}{L_s\nu} \sum_i \hat{N}_i^2 + E_{\text{intra}} \sum_i \hat{N}_i^2 + E_C \hat{N}_1 \hat{N}_2. \quad (4)$$

E_{intra} is an energy scale from the intrachannel Coulomb interaction [the second term of Eq. (1)]. The chemical potential of each channel is determined by the average number of electrons in each channel $\langle \hat{N}_i \rangle$.⁷ Let us define an operator $\delta\hat{N}_i$ which describes the fluctuation of electron number around the average value,

$$\delta\hat{N}_i \equiv \hat{N}_i - \langle \hat{N}_i \rangle, \quad \langle \delta\hat{N}_i \rangle = 0. \quad (5)$$

The Hamiltonian for the fluctuations $\delta\hat{N}_i$ can be obtained by inserting Eq. (5) into Eq. (4) and discarding constant terms,

$$\hat{H}_0 = \sum_{i=1,2} \mu_i \delta\hat{N}_i + E_C \delta\hat{N}_1 \delta\hat{N}_2, \quad (6)$$

where a contribution from $E_{\text{intra}} \sum_i \hat{N}_i^2$ is neglected since the relative fluctuations between two channels will play a more important role in interference. As will be shown below, with the form of Eq. (6), the time evolution of Klein factors can be determined exactly.

Bosonization of electron and quasiparticle operators. The bosonized expression for the electron operator in the edge state of Laughlin quantum Hall liquid at filling fraction ν is given by^{11,13,14} ($i=1,2$)

$$\Psi_{e,i}(x) = \frac{1}{(2\pi a)^{1/2}} F_{e,i} e^{-i2\pi\hat{N}_i x/L_s\nu} e^{-i\phi_i(x)/\nu}, \quad (7)$$

where $F_{e,i}$ is the Klein factor which implements the Fermi statistics of electron operators of different species. It satisfies the following relations:

$$\begin{aligned} [\hat{N}_i, F_{e,j}] &= -F_{e,j} \delta_{ij}, \quad F_i F_i^\dagger = 1, \\ \{F_{e,i}, F_{e,j}\} &= 0, \quad \{F_{e,i}, F_{e,j}^\dagger\} = 0, \quad i \neq j. \end{aligned} \quad (8)$$

Using the bosonization formula of Ref. 15 (with appropriate changes of notations), Eq. (8) can be explicitly realized as follows:

$$\begin{aligned} F_{e,1} &= e^{i\pi(\hat{N}_1 + \hat{N}_2)/2\nu} e^{-i\theta_1/\nu}, \\ F_{e,2} &= e^{-i\pi(\hat{N}_1 + \hat{N}_2)/2\nu} e^{-i\theta_2/\nu}, \end{aligned} \quad (9)$$

where $\theta_{1,2}$ are the operators with zero momentum which are dual to \hat{N}_i in the following sense:

$$[\theta_i, \hat{N}_j] = +i\nu \delta_{ij}. \quad (10)$$

The bosonized expression of the quasiparticle operator is

$$\Psi_{q,i}(x) = \frac{1}{(2\pi a)^{1/2}} F_{q,i} e^{-i2\pi\hat{N}_i x/L_s} e^{-i\phi_i(x)}. \quad (11)$$

We could not find the explicit expression for the Klein factors of quasiparticle operator in literature. Based on the idea that the quasiparticle behaves like a fraction of an electron, we can take the $(1/\nu)$ th root (recall $1/\nu$ is an integer) of Eq. (9), thus leading to

$$\begin{aligned} F_{q,1} &= e^{i\pi(\hat{N}_1 + \hat{N}_2)/2} e^{-i\theta_1}, \\ F_{q,2} &= e^{-i\pi(\hat{N}_1 + \hat{N}_2)/2} e^{-i\theta_2}. \end{aligned} \quad (12)$$

The validity of Eq. (12) can be confirmed by the fact that $F_{q,1}$ and $F_{q,2}$ satisfy the following commutation relations of fractional statistics:

$$[\hat{N}_i, F_{q,j}] = -\nu F_{q,j} \delta_{ij}, \quad F_{q,1} F_{q,2} = e^{-i\pi\nu} F_{q,2} F_{q,1}. \quad (13)$$

III. TIME EVOLUTION OF KLEIN FACTORS

The time evolution of Klein factors under the action of the Hamiltonian (6) can be determined exactly. We use the following operator identity.¹³ Let A, B, D be some operators satisfying $[A, B] = DB$ and $[A, D] = [B, D] = 0$. Then for arbitrary function $f(A)$ of the operator A , we have

$$f(A)B = Bf(A + D). \quad (14)$$

Identifying

$$A \rightarrow \hat{H}_0, \quad B \rightarrow F_{e,1}, \quad D = -\mu_1 - E_C \delta\hat{N}_2, \quad (15)$$

we find [with $f(A) = e^{iAt}$]

$$F_{e,1}(t) = F_{e,1}(t=0) e^{-i\mu_1 t - iE_C \delta\hat{N}_2 t}. \quad (16)$$

The dependence of $F_{e,1}(t)$ on $\delta\hat{N}_2$ gives rise to the additional time dependence for correlation functions, which is not present in Ref. 7. This time dependence will make the visibility exhibit features around the energy scale E_C [see Eq. (36)]. Similarly,

$$F_{e,2}(t) = F_{e,2}(t=0) e^{-i\mu_2 t - iE_C \delta\hat{N}_1 t}. \quad (17)$$

As for quasiparticle operators, we have

$$\begin{aligned} F_{q,1}(t) &= F_{q,1}(t=0) e^{-i\nu\mu_1 t - i\nu E_C \delta\hat{N}_2 t}, \\ F_{q,2}(t) &= F_{q,2}(t=0) e^{-i\nu\mu_2 t - i\nu E_C \delta\hat{N}_1 t}. \end{aligned} \quad (18)$$

IV. TUNNELING CURRENTS

The tunneling current operator is given by (electron tunneling is assumed)

$$\begin{aligned}\hat{I} &= (-e) \frac{d\hat{N}_2}{dt} = \left(-\frac{ie}{\hbar}\right) [\hat{H}_{\text{total}}, \hat{N}_2] \\ &= \left(-\frac{ie}{\hbar}\right) (t_a \Psi_1^\dagger(0) \Psi_2(0) + t_b \Psi_1^\dagger(l_1) \Psi_2(l_2) - \text{H.c.}),\end{aligned}\quad (19)$$

where \hat{H}_{total} is the sum of Eqs. (1)–(3). A straightforward time-dependent perturbation in $t_{a,b}$ yields⁷

$$\begin{aligned}I &\equiv \langle \hat{I}(t=0) \rangle = -\frac{e}{\hbar} \int_{-\infty}^0 dt \left[\sum_{k=1}^4 X_k(t) + \text{H.c.} \right], \\ X_1(t) &= |t_a|^2 \langle [\Psi_1^\dagger(0,0) \Psi_2(0,0), \Psi_2^\dagger(0,t) \Psi_1(0,t)] \rangle, \\ X_2(t) &= |t_b|^2 \langle [\Psi_1^\dagger(l_1,0) \Psi_2(l_2,0), \Psi_2^\dagger(l_2,t) \Psi_1(l_1,t)] \rangle, \\ X_3(t) &= t_a t_b^* \langle [\Psi_1^\dagger(0,0) \Psi_2(0,0), \Psi_2^\dagger(l_2,t) \Psi_1(l_1,t)] \rangle, \\ X_4(t) &= t_a^* t_b \langle [\Psi_1^\dagger(l_1,0) \Psi_2(l_2,0), \Psi_2^\dagger(0,t) \Psi_1(0,t)] \rangle,\end{aligned}\quad (20)$$

where the expectation value should be taken with respect to the eigenstates of the Hamiltonian $\mathbf{H}_{\text{intra}} + \mathbf{H}_C$ [Eqs. (1) and (3)]. For the quasiparticles, the current I must be multiplied by ν . The visibility of interference originates from the correlation functions $X_3(t)$ and $X_4(t)$.

Each correlation function $X_i(t)$ can be factorized into the product of the nonzero-mode (also called oscillators) contribution and the zero-mode contribution. Furthermore, the nonzero-mode contribution is a product of that of channels 1 and 2, since the Hamiltonian of the nonzero modes of channels 1 and 2 commutes each other. The nonzero-mode contribution is identical with that of Ref. 7 and the details are omitted. The zero-mode contribution, however, cannot be expressed as a product of the contribution from each channel, since the zero modes of two channels are coupled by the Hamiltonian (3).

In spite of this noncommutativity, the zero-mode contribution can be computed exactly owing to the results (16) and (18). For example, the zero-mode contribution of $X_1(t)$ in the case of the quasiparticle tunneling is given by

$$\langle F_{q_1}^\dagger(0) F_{q_2}(0) F_{q_2}^\dagger(t) F_{q_1}(t) \rangle = e^{i\nu(\mu_2 - \mu_1)t} e^{-i\nu^2 E_c t}. \quad (21)$$

To obtain Eq. (21), we insert the results (16) and (18) for the time-dependent Klein factors, then employ the commutation relation between Klein factor and the zero-mode operator $\delta\hat{N}_i$, and use $\langle \delta\hat{N}_i \rangle = 0$.

Using complex conjugation properties of X_i , it can be shown that (I_0 and I_Φ defined in an obvious way)

$$I = I_0 + I_\Phi = -\frac{e}{\hbar} \int_{-\infty}^{\infty} dt [Y_0(t) + 2 \text{Re} Y_\Phi(t)]. \quad (22)$$

The visibility is basically determined by the flux-dependent conductance $\sigma_\Phi = dI_\Phi / dV (eV = \mu_2 - \mu_1)$. For the

electron tunneling, the flux-independent part is

$$\begin{aligned}Y_0^{(e)}(t) &= \frac{1}{(2\pi a)^2} (|t_a|^2 + |t_b|^2) e^{i(\mu_2 - \mu_1)t} \\ &\quad \times [g_e^2(x=0, -t) e^{-iE_c t} - \text{H.c.}],\end{aligned}\quad (23)$$

and the flux-dependent part is

$$\begin{aligned}Y_\Phi^{(e)}(t) &= \frac{t_a t_b^*}{(2\pi a)^2} e^{i(\mu_2 - \mu_1)t} e^{2\pi i((N_2)l_2 - (N_1)l_1)/\nu L_s} \\ &\quad \times [g_e(-l_1, -t) g_e(-l_2, -t) e^{-iE_c t} - \text{H.c.}].\end{aligned}\quad (24)$$

$g_e(x, t)$ is the nonzero-mode contribution for the one-electron Green's function, which has been obtained in Ref. 7. At $T=0$,

$$\begin{aligned}c_e(x, t) &= \frac{1}{\nu} \int_0^\infty \frac{dq}{q} e^{-aq} [1 - \cos(qx + \omega_q t)], \\ s_e(x, t) &= \frac{1}{\nu} \int_0^\infty \frac{dq}{q} e^{-aq} \sin(qx + \omega_q t),\end{aligned}\quad (25)$$

where the frequency of nonzero modes is given by

$$\omega_q = q \left[v_0 + v_c \ln \frac{\zeta}{qa} \right], \quad v_c = \frac{\nu e^2}{2\pi \hbar \epsilon}, \quad (26)$$

where $qa \leq 1$ is assumed and $\zeta \approx 1.13$ is a numerical constant. The logarithmic factor of Eq. (26) comes from the intrachannel Coulomb interaction matrix element. The major difference from those of Ref. 7 is the presence of the factor $e^{\pm iE_c t'}$ which originates from the interchannel LRCI. For quasiparticle tunneling, the following modifications are to be made:

$$\begin{aligned}E_c &\rightarrow \nu^2 E_c, \quad (\mu_2 - \mu_1) \rightarrow \nu(\mu_2 - \mu_1), \\ c_e(x, t) &\rightarrow \nu^2 c_e(x, t), \quad s_e(x, t) \rightarrow \nu^2 s_e(x, t).\end{aligned}\quad (27)$$

We note that the energy scale E_c enters in such a way that it is not a mere additive renormalization of the chemical potential. Also, in higher $2n$ th order expansion in tunneling amplitude t_a, t_b , a factor $e^{\pm i\nu E_c t'}$ will emerge since this factor is generated by the commutation of Klein factors. Recall that the Klein factors accompany the tunneling amplitude in Eq. (2). Thus, one can expect some features of visibility which is due to $g_e(x, t)$ will appear being centered at energies nE_c with decreasing magnitude. In the second-order expansion of $t_{a,b}$, the visibility (or equivalently flux-dependent conductance σ_Φ) is found to exhibit nonmonotonic behavior.⁷ Let us try to understand the origin of such behavior in a more analytic way.

V. ASYMPTOTIC BEHAVIOR OF THE ONE-ELECTRON GREEN'S FUNCTION

The correlation function $g_e(x, t)$ cannot be evaluated in a closed form, so that an analytic form of asymptotic behavior will be of great help in understanding the nonmonotonic de-

pendence of conductance on bias. First of all, we note that the equal-time Green's function $g_e(x, t=0)$ is independent of interactions. However, we are mostly interested in the long-time limit, so that we focus on the domain $|x/v_c t| \ll 1$. We write $g_e(x, t)$ in the following form:

$$g_e(x, t) = \exp[-\text{const} + I(x, t)],$$

$$I(x, t) = \frac{1}{\nu} \int_0^\infty \frac{dq e^{-aq}}{q} e^{-i\varphi_q(x, t)}, \quad (28)$$

where $\text{const} = \frac{1}{\nu} \int_0^\infty \frac{dq e^{-aq}}{q}$ is a (infinite) constant, and

$$\varphi_q(x, t) = qx + \omega_q t \quad (29)$$

is a phase function. In fact, the integral $I(x, t)$ diverges at $q=0$ and the divergence is cancelled by the above constant. In spite of this divergence, the form of Eq. (28) is more preferable for the asymptotic analysis. To avoid the divergence at $q=0$, we employ the technique of dimensional regularization:¹⁶ replace the factor $1/q$ of Eq. (28) by $1/q^{1-\alpha}$ with $\alpha > 0$ and take the limit $\alpha \rightarrow 0$ limit and extract the finite contribution. An infinity which appears in the extraction is cancelled by the infinite constant mentioned above.

In the long time/distance limit, the phase $\varphi(x, t)$ becomes very large in the generic domain then we can use the stationary phase approximation.¹⁰ In general, there exist two contributions to the asymptotic behavior of the integral of the type of $I(x, t)$: one from the ends of integral interval ($q=0$ and $q=\infty$, clearly the contribution from $q=\infty$ is negligible) and the other from the stationary point(s) where $d\varphi_q(x, t)/dq=0$.

The contribution from the end point at $q=0$ can be obtained by the integration by parts method.¹⁰ Define a variable u as follows:

$$u = u(q) = q \frac{x}{t} + \omega_q = qv_c \ln\left(\frac{\eta}{qa}\right), \quad (30)$$

where $\eta = \zeta e^{[(x/t)+v_0]/v_c}$. Changing the integration variable from q to u , the relevant integral becomes

$$\int_0^\infty du \left| \frac{dq}{du} \right| \frac{e^{-aq}}{[q(u)]^{1-\alpha}} e^{-itu}. \quad (31)$$

Let us consider the case where $u > 0$. Within logarithmic accuracy, we have $q(u) \sim \frac{u}{v_c} [\ln \frac{\eta v_c}{ua}]^{-1}$ and $\frac{dq}{du} \sim \frac{1}{v_c \ln \frac{\eta v_c}{ua}}$ with $\eta^* = e^{[(x/t)+v_0]/v_c}$. Performing the partial integration along imaginary axis,¹⁰ taking $\alpha \rightarrow 0$ limit, and extracting the finite part, we obtain

$$I_{\text{end}}(x, t) \sim -\ln\left[\frac{v_c t}{a} \ln(\eta v_c t/a)\right] - i \text{sign}(t) \pi/2. \quad (32)$$

The effect of LRCI within each channel is reflected in the double-logarithmic correction $\ln[\ln(\eta v_c t/a)]$. Note that this correction also depends on position l_1, l_2 through η . In terms of the Green's function $g_e(x, t)$, the end-point contribution is roughly $1/[t \ln t]^{1/\nu}$, which evidently cannot cause the non-monotonic behavior of the flux-dependent conductance σ_Φ as shown in Ref. 7.

Next we turn to the contribution from stationary point. The condition for the stationary point is

$$\left. \frac{d\varphi_q(x, t)}{dq} \right|_{q=q_c} = 0 \rightarrow -\frac{x}{t} = \left. \frac{d\omega_q}{dq} \right|_{q=q_c} \sim v_c \ln \frac{\bar{\zeta}}{q_c a}, \quad (33)$$

where $\bar{\zeta} = \zeta e^{(v_0/v_c)-1}$, and q_c is determined by x/t . For the condition (33) to be satisfied for the very long-time limit (namely, $|x/v_c t| \ll 1$), there should be a point where $\left. \frac{d\omega_q}{dq} \right|_{q=q_c} = 0$. In other words, the frequency should attain a local maximum or minimum at finite momentum. Therefore, if ω_q is a monotonic function of q then there will be no stationary point, so that the nonmonotonic behavior of visibility would not appear. Another dispersion $\omega_q = v_0 q - bq^3$ with $b > 0$ (Ref. 7) which also shows nonmonotonous behavior satisfies the local maximum condition. It is easily seen $\left. \frac{d^2\omega_q(x, t)}{dq^2} \right|_{q=q_c} = -\frac{v_c}{q_c}$. Then the standard stationary phase approximation¹⁰ gives

$$I_{\text{sta}}(x, t) \sim \sqrt{\frac{2\pi}{|t|q_c v_c}} e^{-i\pi/4 \text{sgn } t} e^{-i\varphi_q(x, t)|_{q=q_c}}. \quad (34)$$

In the long-time limit, $|I_{\text{end}}(x, t)| \gg |I_{\text{sta}}(x, t)|$. Were it not for the $1/q$ singularity at $q=0$, the stationary point contribution ($1/\sqrt{t}$) would dominate the singularity-free end-point contribution ($1/t$). Equation (34) explains the oscillating behavior of the Green's function at the long-time tail as found in Fig. 4 of Ref. 7. Now the asymptotic form of the correlation function $g_e(x, t)$ is given by

$$g_e \sim e^{I_{\text{end}} + I_{\text{sta}}} \sim \frac{1}{\left[\frac{v_c t}{a} \ln(\bar{\eta} v_c t/a)\right]^{1/\nu}} (1 + I_{\text{sta}}). \quad (35)$$

VI. DISCUSSIONS

The origin of the nonmonotonic behavior of visibility found in Ref. 7 is the oscillating tail of the one-particle electron Green's function. In this paper, we have found that the oscillating tail is due to the specific momentum dependence of collective excitation. The investigation of the detailed form of the visibility requires the numerical integration,⁷ but the essential features can be understood qualitatively using Eq. (35). Taking the typical time as $t \sim 1/|eV - E_c|$, ($\Delta = \hbar v_c / a$), one can estimate the integral of Eq. (22) to obtain

$$\sigma_\Phi \sim \left[|eV - E_c| \ln \frac{|eV - E_c|}{\Delta} \right]^{(2/\nu)-2} \times \left\{ 1 + \left| \frac{eV - E_c}{\Delta} \right|^{1/2} \cos[\tilde{\varphi}_c(V)] \right\}, \quad (36)$$

where $\tilde{\varphi}_c(V)$ is basically the phase function φ_q evaluated at the stationary point, and it slowly varies as a function of bias. The factor $\cos[\tilde{\varphi}_c(V)]$ is responsible for the nonmonotonic behavior. At integer filling $\nu=1$, the presence of energy scale E_c is not so pronounced because the exponent of prefactor of

Eq. (36) vanishes. Compared to Ref. 7, the most dramatic difference with QH case ($\nu < 1$, for electron tunneling) would be the strong suppression of amplitude near $eV \sim E_c$. As for the quasiparticle tunneling (at $\nu < 1$), the time integral of Eq. (22) diverges in the long-time limit, which implies that the MZI is entering the strong-coupling regime at low temperature. It is well known that the strong-coupling regime of QH point contact is well described by the electron-tunneling picture.¹⁷ At high temperature, the divergence in the long-time limit is cut by $1/T$. Thus as temperature decreases, we can expect a crossover from quasiparticle tunneling to electron tunneling, and at the same time the interchannel Coulomb energy scale changes from $\nu^2 E_c$ to E_c . All these features can be readily checked experimentally.

We have to note that the results of both Ref. 7 and the present paper have been obtained in the second-order perturbation with respect to tunneling amplitudes. As such, both

results do not seem to explain the experimental results of the periodic behavior of visibility. However, the present paper indicates the existence of a series of energy scales nE_c ($n = 1, 2, 3, \dots$), and it clearly suggests that we need the higher-order perturbations to reveal a possible underlying periodic structure of visibility.

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¹⁴We choose the convention of left-moving particles.

¹⁵J. Polchinski, *String Theory* (Cambridge University Press, Cambridge, England, 1998), Vol. 1, p. 239.

¹⁶P. Ramond, *Field Theory: A Modern Primer* (Westview, Colorado, 1990).

¹⁷K. Imura and N. Nagaosa, *Solid State Commun.* **103**, 663 (1997).